On design of Height-Based Terrain-Following Coordinate Used in Atmospheric Numerical Model

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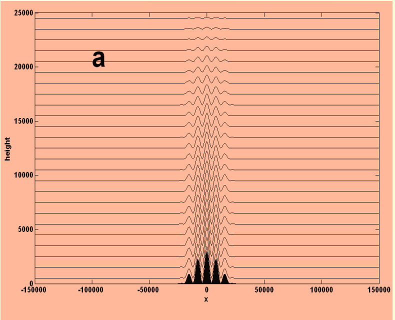
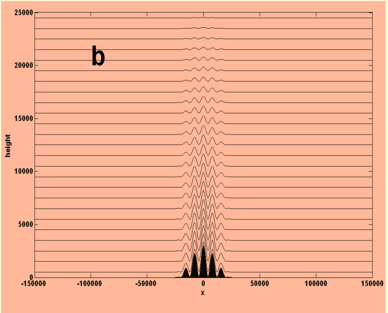
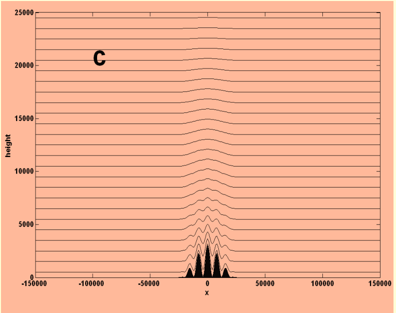
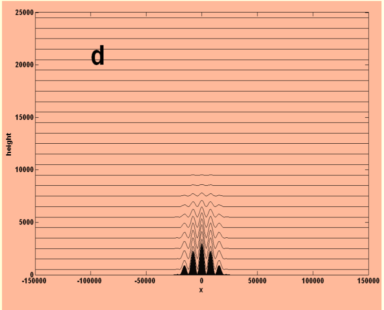








Outline

* 1. Introduction
* 2. New TF vertical coordinates
* 3. 1D idealized tests
* 4. 2D idealized tests
* 5. Conclusions

1 Introduction

Two widely used formulations for TF coordinate

- A Pressure-based TF coordinate

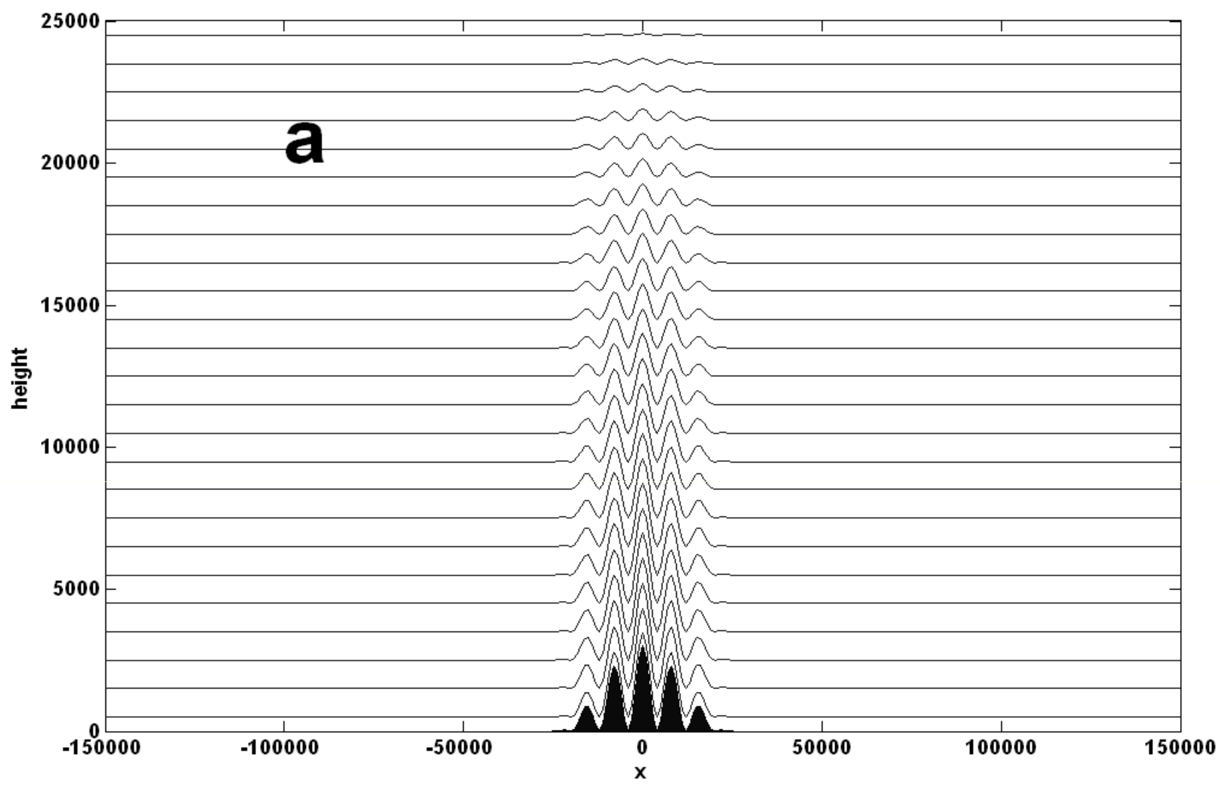
$$\sigma_p = \frac{P - P_T}{P_s - P_T}$$

- A Height-based TF coordinate

$$\hat{z} = Z_T \frac{z - Z_s(x, y)}{Z_T - Z_s(x, y)}$$

- *The TF (Terrain Following) vertical coordinates have been widely used in the current research and operational NWP models.*

The advantages of the terrain-following coordinate



The $\hat{z}$ - coordinate surface on different vertical levels.

$$\hat{z} = Z_T \frac{z - Z_s(x,y)}{Z_T - Z_s(x,y)}$$

(Chen-Sommerville, 1975)

Vertical: height levels;
Horizontal: x-grid point.

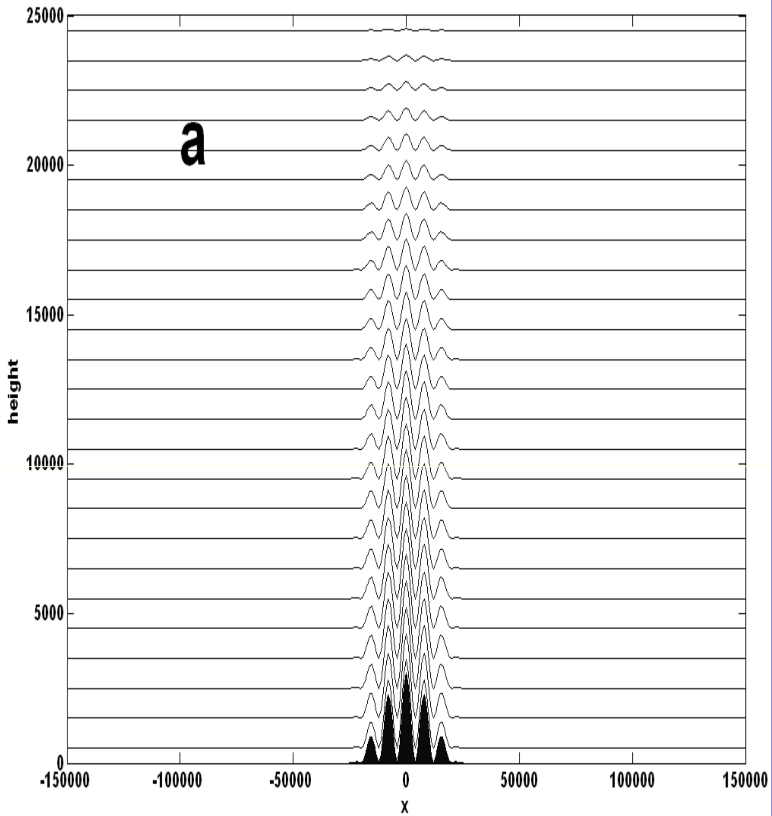
The main advantages for a NWP model using the TF-coordinate:

•(1) It is simple to specify the bottom boundary condition, specially over a complex orography

•(2) It is easy to compute the physical flux between the low atmosphere and the earth surface on a unequal vertical layers.

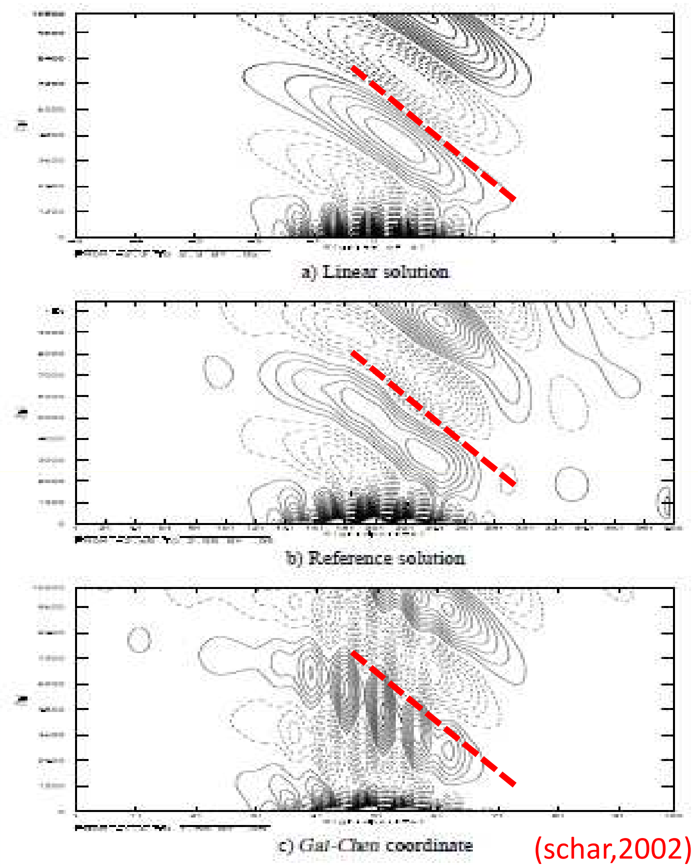
The disadvantages of the terrain-following coordinate

(a)



(G-Chen-Sommerville, 1975)

(b)



(schar,2002)

Disadvantages as followings:

- (1) The sloping coordinate surfaces could inflict significant distortion of the quasi-horizontal motions on the upper levels (en left);
- (2) The deformation of the gravity waves structure when the model resolution is highly increased (on right);
- (3) The errors of PGF's calculation induced by using a TF coordinate (To see followings).

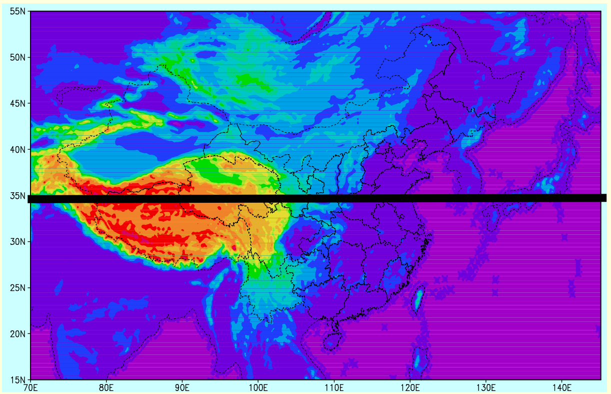
An example: to estimate errors of PGF's calculation

PGF-1 term to PGF-2 terms in a TF coordinate

Calculation of PGF:

$$-\alpha\nabla_z P_s \longrightarrow -\alpha\nabla_{\hat{z}} P + \frac{\alpha}{g}\frac{\partial P}{\partial \hat{z}}\cdot\frac{\Delta Z_{\hat{z}}}{\Delta Z_s}\cdot\nabla_{\hat{z}}\phi_s$$

Model topography:



GRAPES's Meso domain

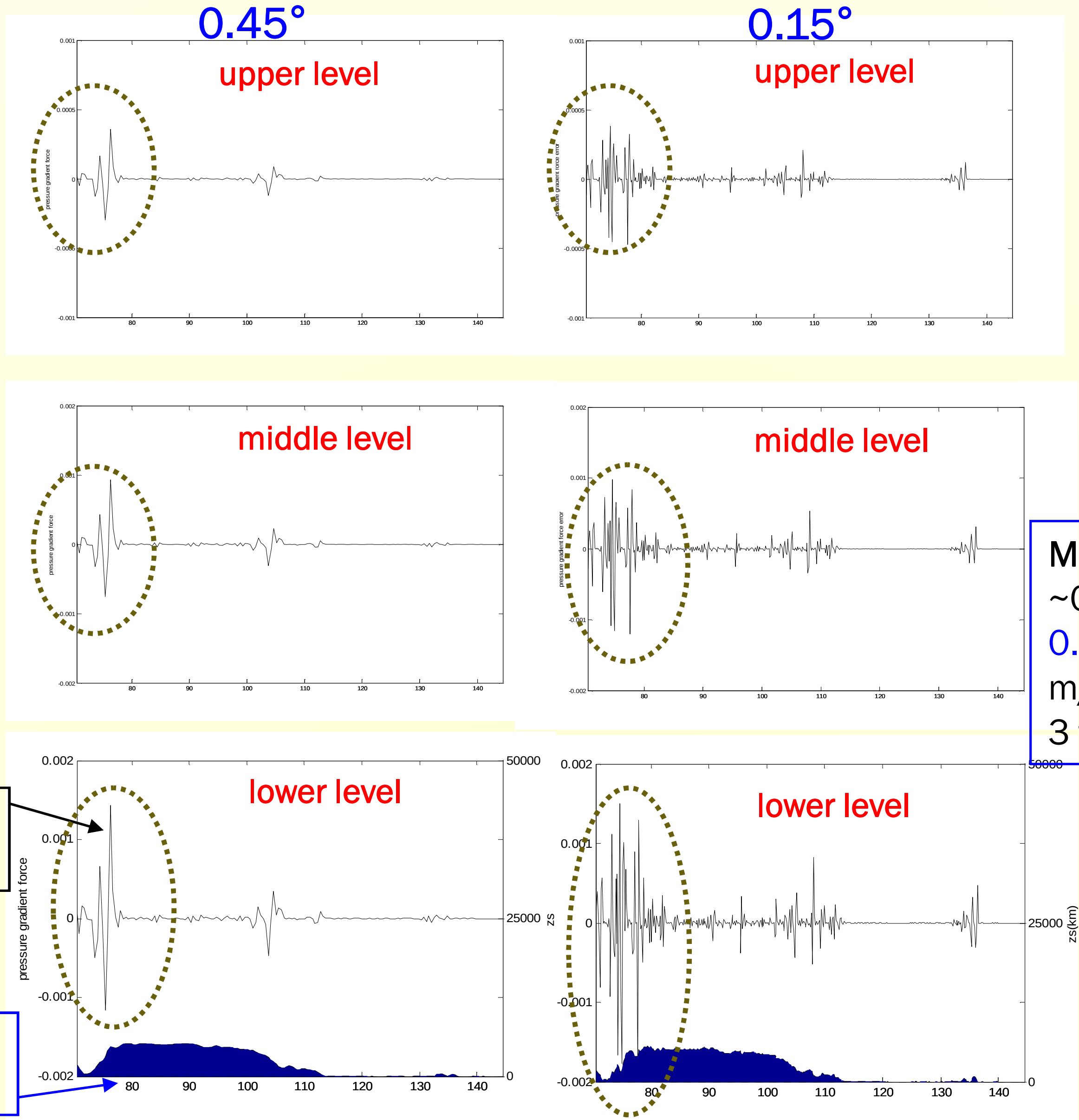
34°N

Reference atmosphere without horizontal changes:

$$\begin{cases} \overline{\Pi} = \exp\left(-\dfrac{gz}{C_p T_0}\right) \\ \overline{\theta} = T_0 \exp\left(\dfrac{gz}{C_p T_0}\right) \end{cases}$$

Inter-comparison to the errors of calculation of PFG with 0.45 °and 0.15 ° resolutions along the 34°N for a rest-atmosphere without horizontal variation.

To estimate the errors of PGF's calculation



Black line: PGF's Errors

Shaded area: model orography

Max-error of w-wind : ~0.15 m/s with 0.15°VS ~0.05 m/s with 0.45°, 3 times higher.

2. New TF vertical coordinates

General formulation

- In order to design a new TF coordinate, we rewrite the formulation of Gal-Chen and Sommerville (1975) as following:

$$\hat{z} = Z_T \frac{z - Z_s(x,y)}{Z_T - Z_s(x,y)}$$

$$\longrightarrow \quad z = \hat{z} + b \cdot Z_s(x,y)$$

with $b = (1 - \frac{\hat{z}}{Z_T})$ It is a decaying coefficient of the coordinate surface with height. It is possible to use different “b” to accelerate the decaying.

2. New TF coordinates

- The different decaying coefficients "b" can be defined as:

G.C.S.

$$b = (1 - \frac{\hat{z}}{Z_T})$$

(Gal-Chen and Sommerville, 1974)

SLEVE1

$$b_h = \frac{\sinh[(Z_T - \hat{z})/h^*]}{\sinh[Z_T / h^*]}$$

SLEVE2

$$b_H = \sum_{i=1}^{2} \frac{\sinh[(Z_T - \hat{z})/h_i^*]}{\sinh[Z_T / h_i^*]}$$

(Schar, 2002)
h*: scale of ref-topography; h^*_1 and h^*_2: large and small-scale of ref-topogr.

COS

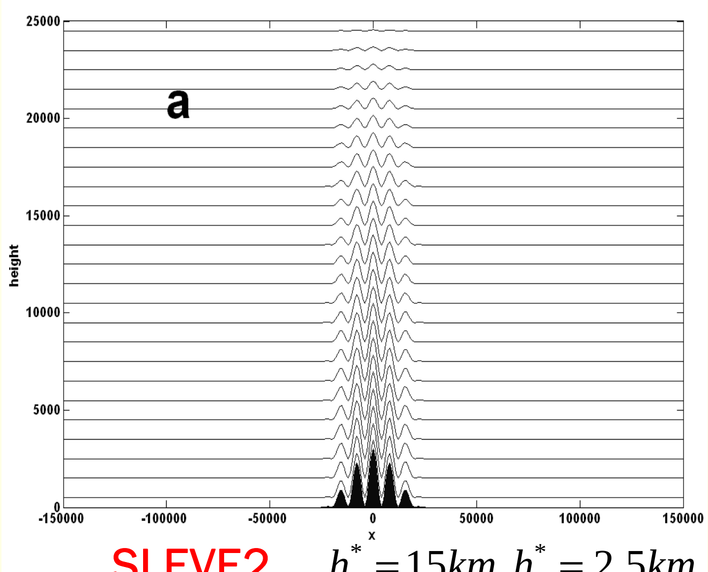
$$b_C = \begin{cases} \left(1 - \frac{\hat{z}}{Z_T}\right) \cdot \left[\cos\left(\frac{\pi}{2}\frac{\hat{z}}{\hat{z}_c}\right)\right]^n & \hat{z} \le \hat{z}_c \\ 0 & \hat{z} > \hat{z}_c \end{cases}$$

(similar to Klemp, 2011)
"n>2": an empirical number; z_c : a reference height from which the coordinate surface becomes horizontal.

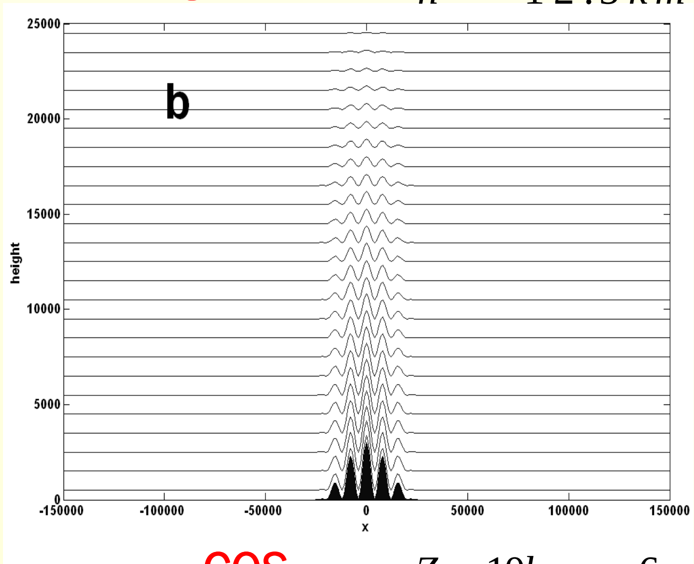
2. New TF coordinates (cont.)

- For GCS-coordinate, $\frac{\partial b}{\partial \hat{z}} = -\frac{\hat{z}}{Z_T} \equiv constant$ means that the coordinate surface is smoothing in a constant rate with height until to the top of model.
- In contrary, for SLEVE1, SLEVE2 and COS coordinates, the decaying coefficient "b_h"(or "b_c") is negatively increased with height. It means that the coordinate surface can be accelerated to be smoothed with height until to the reference-level (z_c).

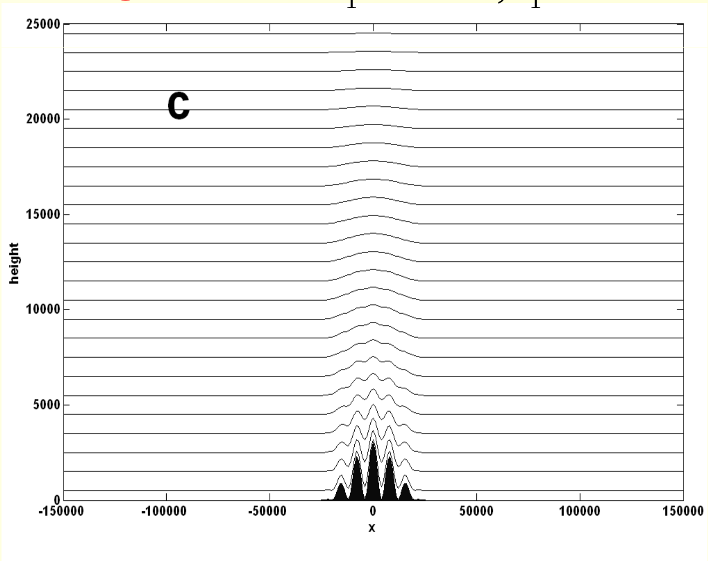
G.C.S



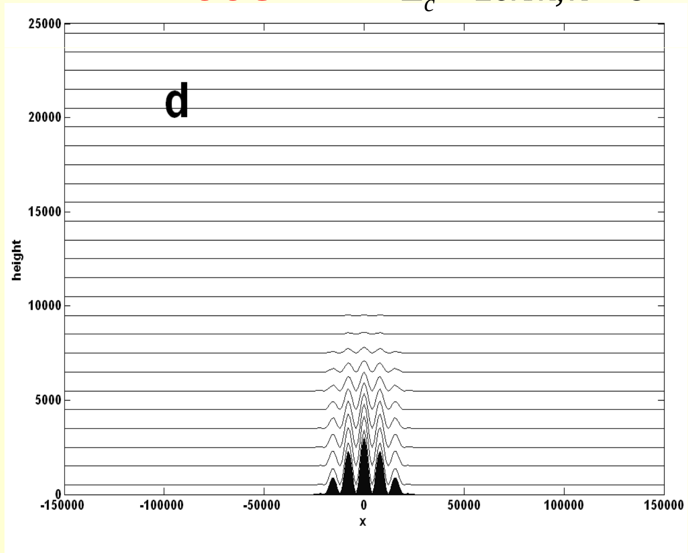
SLEVE1 $h^{*} = 12.5km$



SLEVE2 $h_1^{*} = 15km, h_1^{*} = 2.5km$



COS $Z_c = 10km, n = 6$

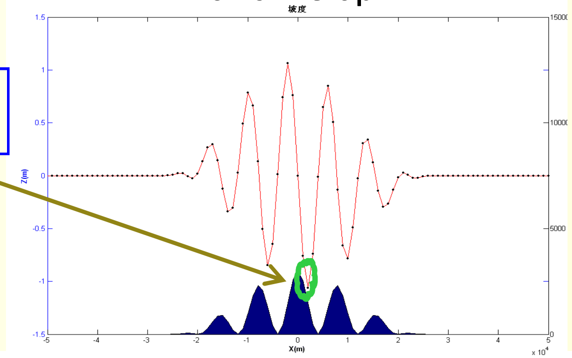


The coordinate surfaces on vertical levels: Vertical: height levels; Horizontal: x-grid point.

Theoretical analyses of the new TF coordinates

Terrain slop

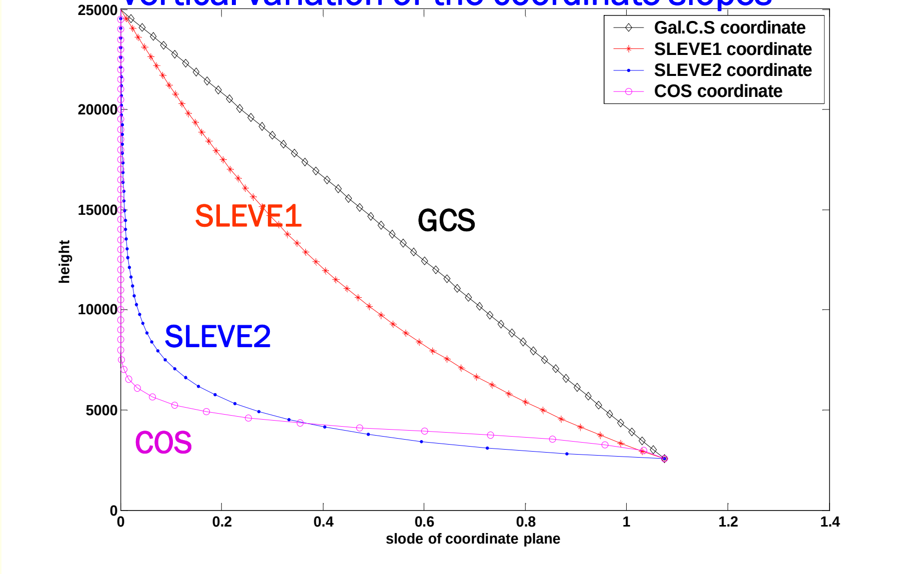
The highest terrain slope



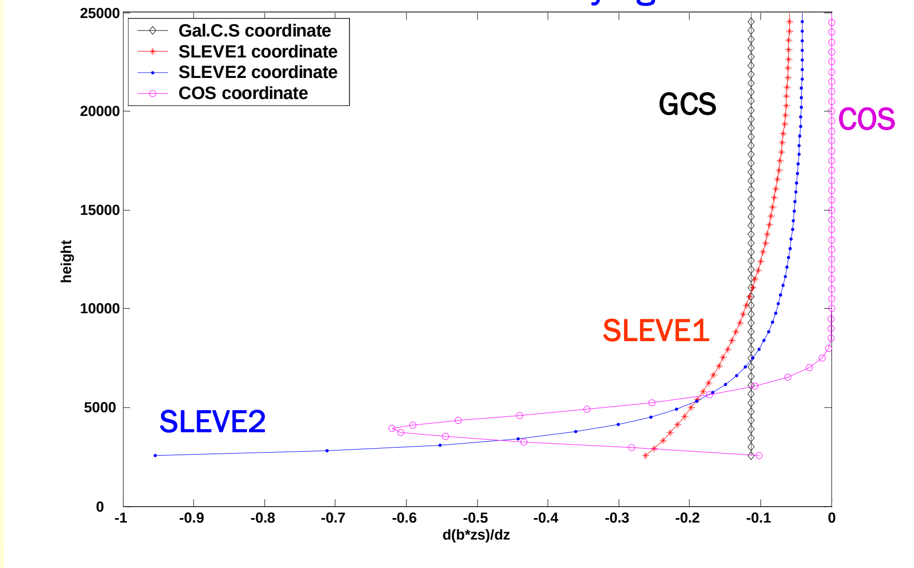
Defining the vertical derivatives of the decaying function $b \bullet Zs$

$$\frac{\partial (b \bullet Zs)}{\partial z}$$

Vertical variation of the coordinate slopes



Vertical variations of the decaying function



（ left ） black for G.C.S, red for SLEVE1, blue for SLEVE2, and purple for COS coordinates, respectively

（ right ） black for G.C.S, red for SLEVE1, blue for SLEVE2, and purple for COS coordinates, respectively

3. 1D idealized tests

(for a rest atmosphere test)

1D test design

•**Test Objective** ： to compare the errors of PGF calculation of four coordinates in rest atmosphere over an artificial terrain.

•**Test design** ：

•Reference rest atmosphere ：

$$\begin{cases} \overline{\Pi} = \exp\left(-\dfrac{gz}{C_p T_0}\right) \\ \overline{\theta} = T_0 \exp\left(\dfrac{gz}{C_p T_0}\right) \end{cases}, \quad g = 9.81, T_0 = 287.0$$

•Classical algorithm used for PGF calculation

$$-C_p \theta \nabla_z \Pi = -C_p \theta \nabla_{\hat{z}} \Pi + C_p \theta J_b \cdot \frac{\partial \Pi}{\partial \hat{z}} \cdot b \cdot \nabla_{\hat{z}} (Z_S(x, y))$$

with $J_b = \dfrac{\partial \hat{z}}{\partial z}$

Specification of the parameters related to TF coordinate experiments

(1) the top of model: $Z_T = 25\ km$

(2) Vertical levels: 50 with 500m of uniform thickness of each layer;

(3) Horizontal resolution: total grid numbers along x-axe: 100000; $\delta x = 1 km$

(4) Terrain function:
a=25km
λ=8km

$$Z_s = \cos^2\left(\frac{\pi \cdot x}{\lambda_s}\right) \cdot z^*$$

$$Z_{s0} = 3km$$

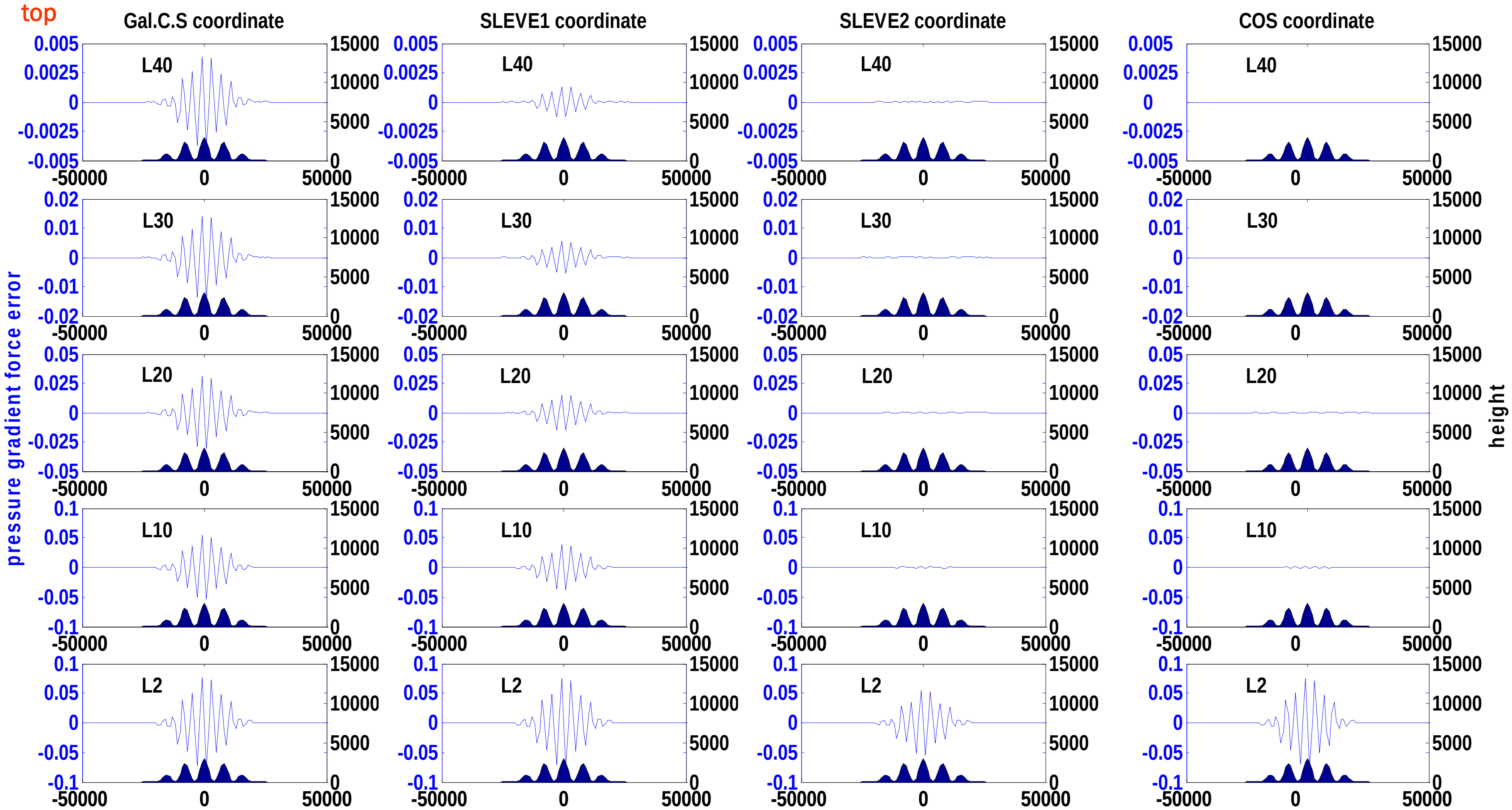
$$z^* = \begin{cases} Z_{s0} \cos^2\left(\frac{\pi \cdot x}{2 \cdot a}\right) & |x| \le 25km \\ 0 & |x| > 25km \end{cases}$$

(5) $h^* = 12.5km$ for SLEVE1; $h_1^* = 15km; \quad h_2^* = 2.5km$ SLEVE1;

$z_c = 10km; \quad n = 6$ for "COS" coordinate

Errors of PGF calculation induced by using TF coordinates

G.C.S	SLEVE1	SLEVE2	COS



On different vertical levels: L2, L10, L20, L30 and L40 from bottom to top

Relatively Reduced Errors: SLEVE1(SLEVE2, COS) against GCS

R.R.E. is defined as:

$$\varepsilon = \frac{E_{Gal} - E_{SLEVE1/SLEVE2/COS}}{E_{Gal}} \times 100\%$$

Vertical levels	SLEVE1	SLEVE2	COS
L40	67%	99%	100%
L30	62%	99%	100%
L20	51%	99%	99%
L10	31%	95%	75%
L2	4%	30%	2%

4. 2D idealized tests

(Idealized advection tests)

2D test design

•**Test Objective** : To quantify the PGF's errors in a two dimensional advection of air mass over a simple topographic obstacle.

•**Test design** : The transport equation of air mass is described in a conservative flux form:

$$\frac{\partial(J_b^{-1}\rho)}{\partial t}+\frac{\partial(J_b^{-1}u\rho)}{\partial x}+\frac{\partial(J_b^{-1}\hat{w}\rho)}{\partial \hat{z}}=0$$

The above equation can be discretized by using explicit time stepping with centered finite differences in space and time (leapfrog) on a staggered Arakawa C grid :

$$q=J_b^{-1}\rho \quad , \quad F=J_b^{-1}u\rho \quad , \quad G=J_b^{-1}\hat{w}\rho$$

$$F_{i-1/2,k}=u^{-}q_{i,k}^{n}+u^{+}q_{i-1,k}^{n} \quad , \quad u^{+}=\max(u,0), u^{-}=\min(u,0)$$

$$G_{i,k-1/2}=\hat{w}^{-}q_{i,k}^{n}+\hat{w}^{+}q_{i,k-1}^{n} \quad , \quad \hat{w}^{+}=\max(\hat{w},0), \hat{w}^{-}=\min(\hat{w},0)$$

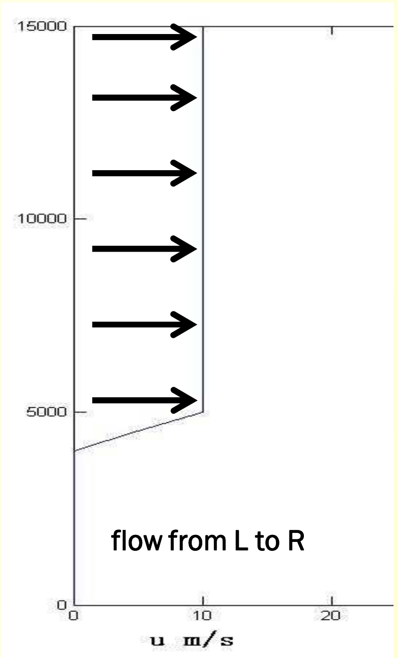
2D test design (cont.)

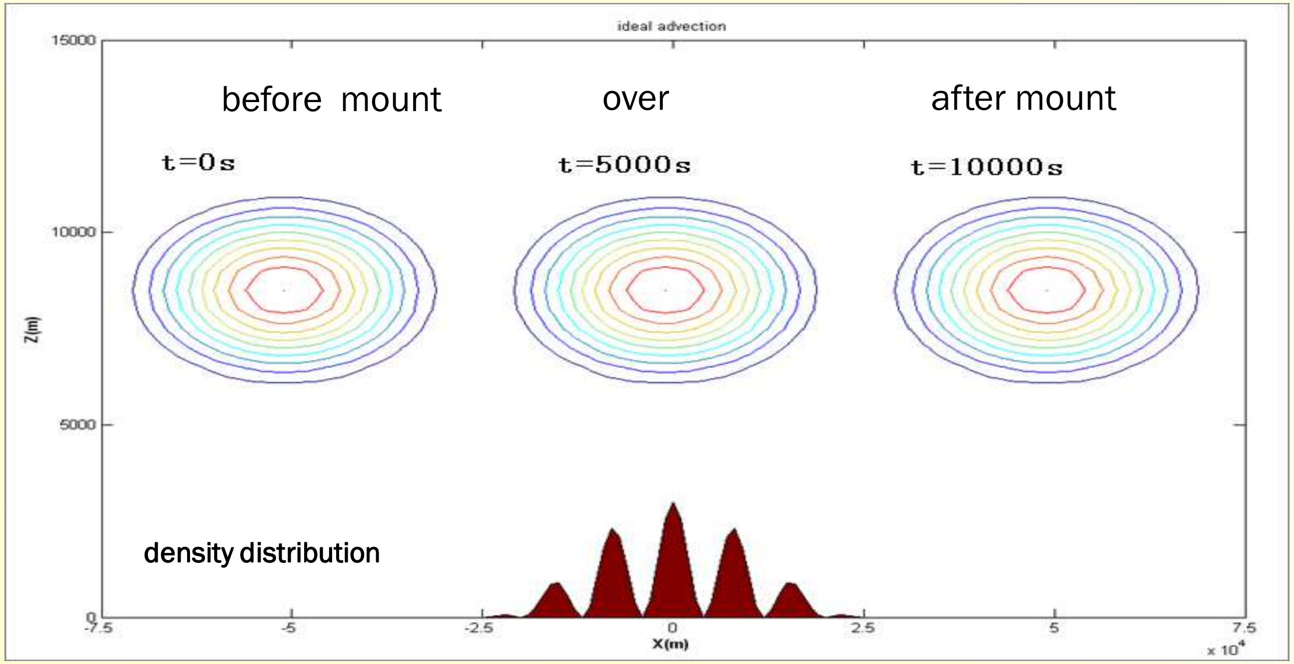
Initial wind:

$$u(z) = 10 \cdot \begin{cases} 1 & 5km \le z \\ \sin^2\left(\frac{\pi}{2} \cdot \frac{z - z_1}{z_2 - z_1}\right) & 4km \le z \le z \\ 0 & z \le 4km \end{cases}$$

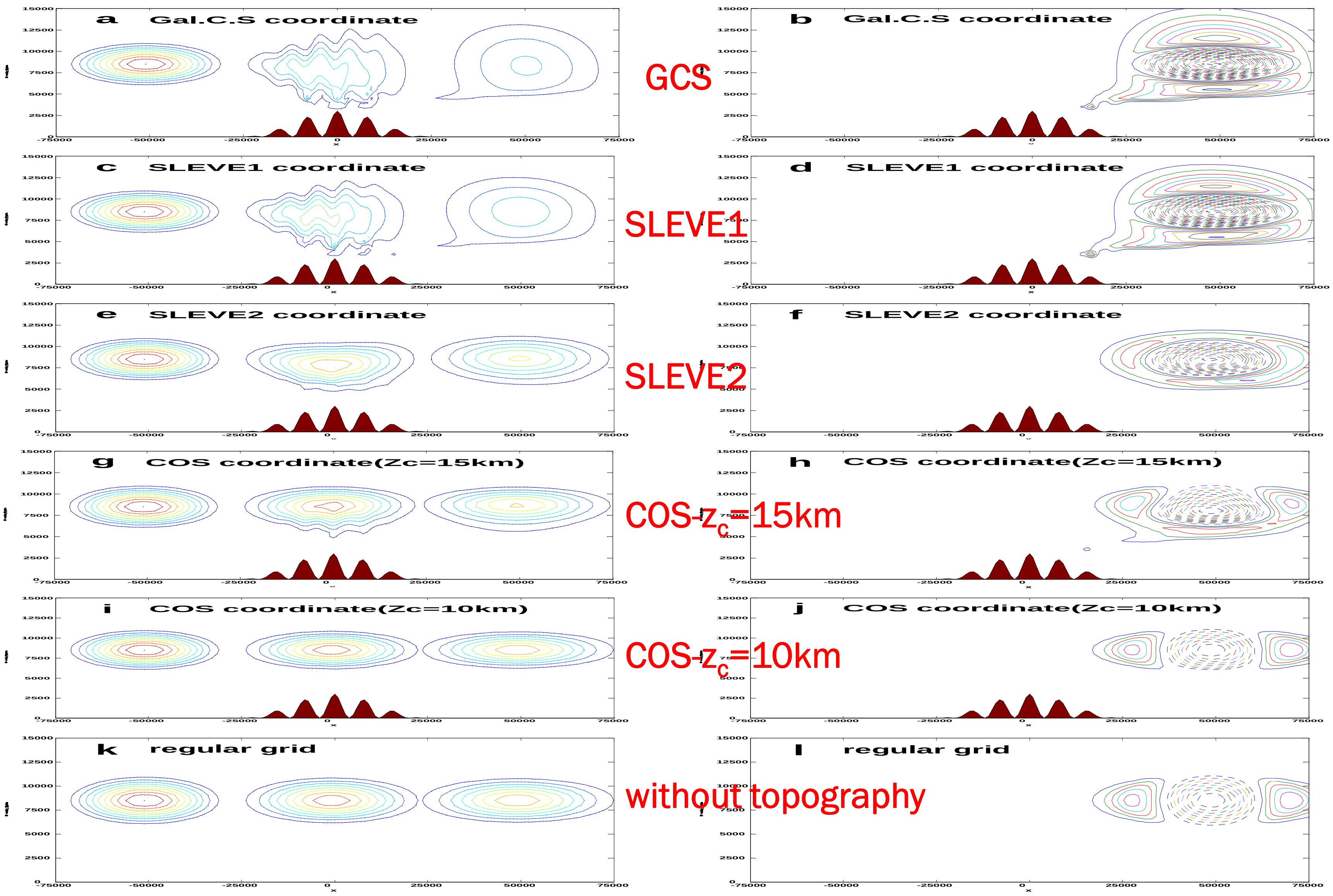
Analysis density distribution :

$$\rho(x,z) = \rho_0 \cdot \begin{cases} \cos^2\left(\frac{\pi \cdot r}{2}\right) & r \le 1 \\ 0 & r > 1 \end{cases}, \quad r = \sqrt{\left(\frac{x - x_0}{R_x}\right) + \left(\frac{z - z_0}{R_z}\right)}$$





Advection test : air mass advects through a topographic obstacle



left : density distribution at 0s,5000s,10000s

right : the errors at 10000s after mountain

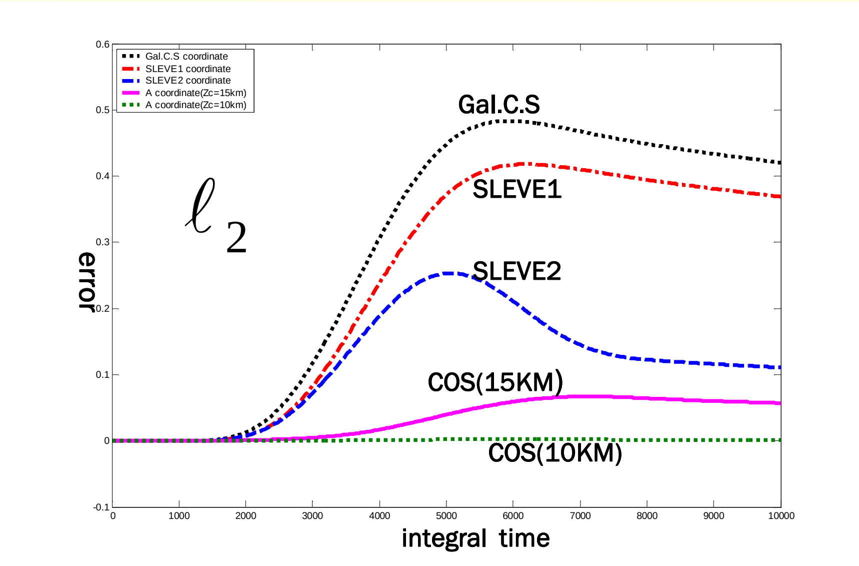
Quantitatively comparing the errors of the simulations

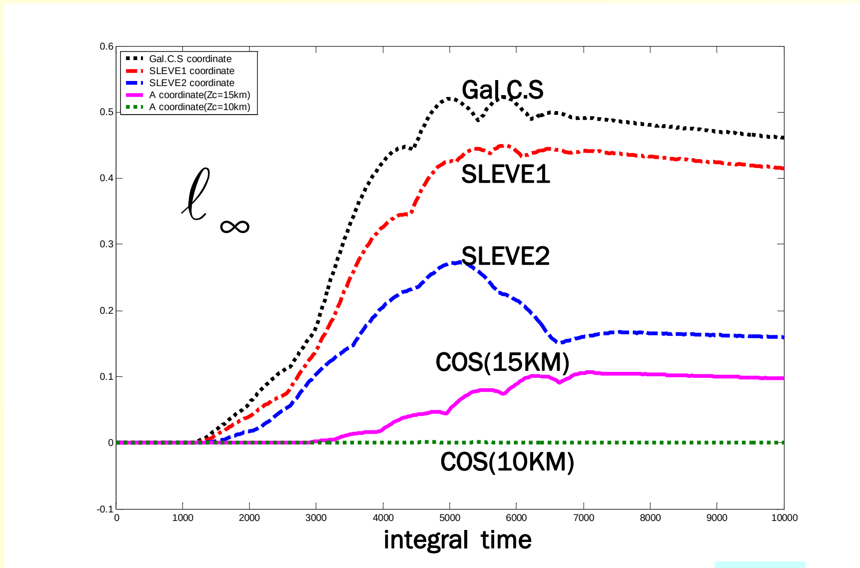
- Defining two parameters as following, according to Williamson (et.al,1992)

$$\ell_2 = \left\{ \frac{\left\{ \sum_{k=1}^{m} \sum_{i=1}^{n} \left[(\rho_{i,k})_{num} - (\rho_{i,k})_{ana} \right]^2 \right\}^{\frac{1}{2}}}{\left\{ \sum_{k=1}^{m} \sum_{i=1}^{n} \left[(\rho_{i,k})_{ana} \right]^2 \right\}^{\frac{1}{2}}} \right\}_{SLEVE1, SLEVE2, COS} - \left\{ \frac{\left\{ \sum_{k=1}^{m} \sum_{i=1}^{n} \left[(\rho_{i,k})_{num} - (\rho_{i,k})_{ana} \right]^2 \right\}^{\frac{1}{2}}}{\left\{ \sum_{k=1}^{m} \sum_{i=1}^{n} \left[(\rho_{i,k})_{ana} \right]^2 \right\}^{\frac{1}{2}}} \right\}_{without\ terrain}$$

$$\ell_\infty = \left\{ \frac{MAX \left| (\rho_{i,k})_{num} - (\rho_{i,k})_{ana} \right|}{MAX \left| (\rho_{i,k})_{ana} \right|} \right\}_{SLEVE1, SLEVE2, COS} - \left\{ \frac{MAX \left| (\rho_{i,k})_{num} - (\rho_{i,k})_{ana} \right|}{MAX \left| (\rho_{i,k})_{ana} \right|} \right\}_{without\ terrain}$$

$(\rho_{i,k})_{num}$ is numerical solution $(\rho_{i,k})_{ana}$ is analytical solution





left : temporal evolution of ℓ_2 **right : temporal evolution of** ℓ_∞

5. Conclusions

5.1 conclusion

- **It can be summarized for this study as following:**
- (1) The significant numerical errors of PGF calculation can found from bottom to top of the model with a traditional TF coordinate such as GCS (Gal-Chen and Sommerville, 1975).
- (2) To design a new smoothed TF coordinate, it is essential to modify the decaying coefficient “b”:

$$z = \hat{z} + b \cdot Z_s(x, y)$$

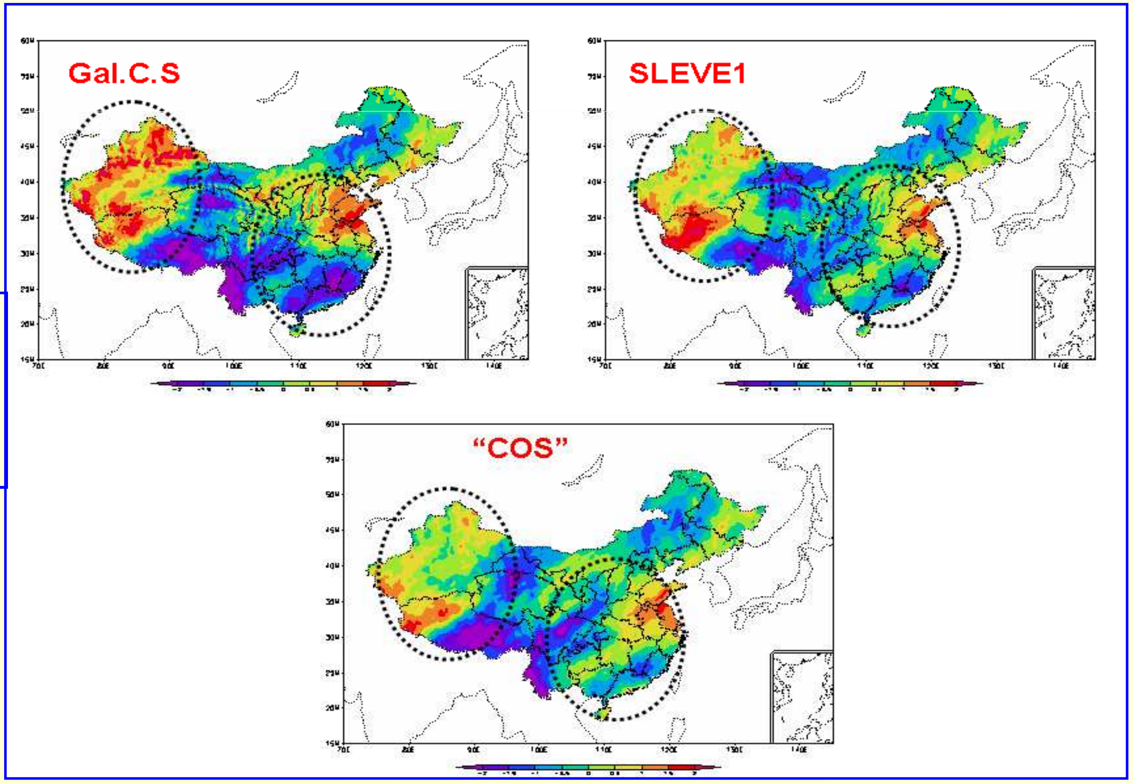
5.1 conclusion (cont.)

- (3) The 1D tests with a rest-atmosphere showed that the new TF coordinates SLEVE1 and SLEVE2 (Schar, 2002), “COS” coordinate are useful in reducing the numerical errors of PGF calculation. More than 99% of the errors could be reduced from the 20th level (~10km) with SLEVE2 and COS coordinates.
- (4) For the idealized advection tests, the new TF coordinates, more or less, could improve the conservative advection of air mass through a simple topography in comparison to the traditional GCS coordinate.
- (5) The new TF coordinate “COS” gave the best results in reduction of the numerical errors of PGF, and in conservative advection of air mass in comparison to other TF coordinates.

5.2 next step

- The traditional GCS coordinate (Gal-Chen and Sommerville, 1995) was currently used in new generation of NWP system GRAPES.
- We are going to incorporate the 3 new TF coordinates in GRAPES (regional and global). The preliminary results with regional GRAPES are quite encouraging:

Monthly mean of 24h forecast of geopotential height at 100hPa



Thank you !